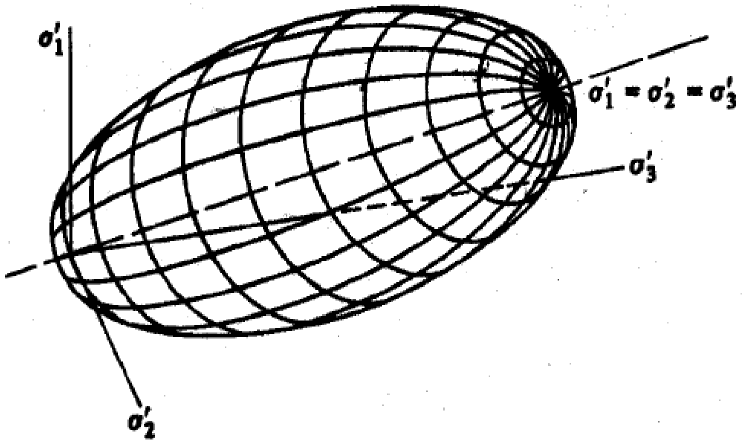


Geomechanics

LECTURE 7

MODIFIED CAM-CLAY MODEL

Dr. ALESSIO FERRARI

Laboratory of soil mechanics – Fall 2024

28.10.2024

Access the QUIZ

https://etc.ch/Py4h

Geomechanics
General Course overview

Semester

Topics

Basic concepts

Classical constitutive framework

- Elasticity - Linear & Non-linear
- Plasticity Perfect-plasticity
- Hardening elasto-plasticity
- Critical state concept
- Modified Cam-Clay

Practical aspects

- Geomechanics in Practice
- Numerical modelling
- Site visit
- In-situ stress
- Course Project
- Retaining structures

Advanced constitutive models

- Unsaturated behavior
- THM behaviour of geomaterials
- Bio-cementation
- Time-dependent behaviour

Content

- Modified Cam-Clay model
  - Elastic behaviour
  - Yield function
  - Plastic potential
  - Hardening rule
- MCC prediction
- Conclusion

Modified Cam Clay model (MCC)

ELASTIC BEHAVIOUR

YIELD FUNCTION

PLASTIC POTENTIAL

HARDENING RULE

MCC - Introduction

Originally developed at University of **Cambridge** for saturated **clay**

Original Cam-Clay – Roscoe et al. 1958

Further works on the original model yielded a modified version

Modified Cam-Clay - Schofield & Worth 1968

The model is based on:

- **Critical state concept**
- **Strain hardening elasto-plasticity**

MCC – Stress and strain framework

Triaxial stress-strain

Mean pressure: $p' = \dfrac{\sigma_1 + 2\sigma_3}{3} = \dfrac{J_1}{3}$

Volumetric strain: $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$

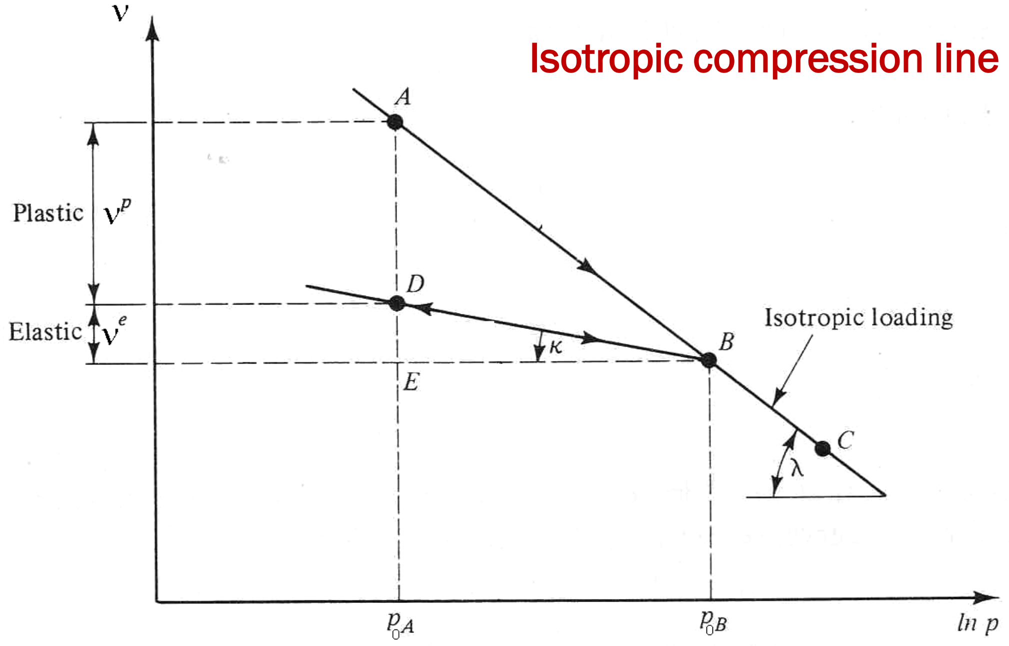
Deviatoric stress: $q = \sigma_1 - \sigma_3 = \sqrt{3\mathrm{J}_{2\mathrm{D}}}$

Deviatoric strain: $\varepsilon_d = \dfrac{2}{3}(\varepsilon_1 - \varepsilon_3)$

Components of hardening elastoplastic model

(i) elastic behavior
(ii) yield surface
(iii) plastic potential and flow rule
(iv) hardening rule

MCC – Elastic behaviour



$\boxed{\Delta v = \Delta e}$ ➡ $\Delta e^e = e_D - e_E = -\kappa \ln\left(\frac{p'_B}{p'_A}\right)$ ➡ $de^e = -\kappa \frac{dp'}{p'}$

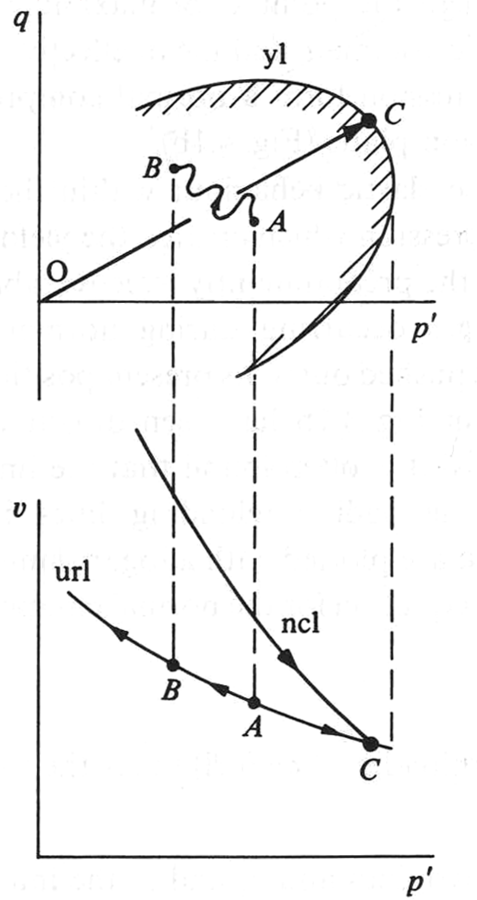
Recall:

$$d\varepsilon_v = -\frac{d\upsilon}{\upsilon} = -\frac{de}{1+e}$$ ➡

$$d\varepsilon_v^e = -\frac{de^e}{1+e} = \frac{\kappa}{1+e}\frac{dp'}{p'}$$

$$d\varepsilon_d^e = \frac{dq}{3G}$$

yl: yield surface corresponding to ncl



ncl : Normal compression line

url: unloading-reloading line

Wood, 1990

MCC – Elastic part - Summary

From isotropic linear elasticity

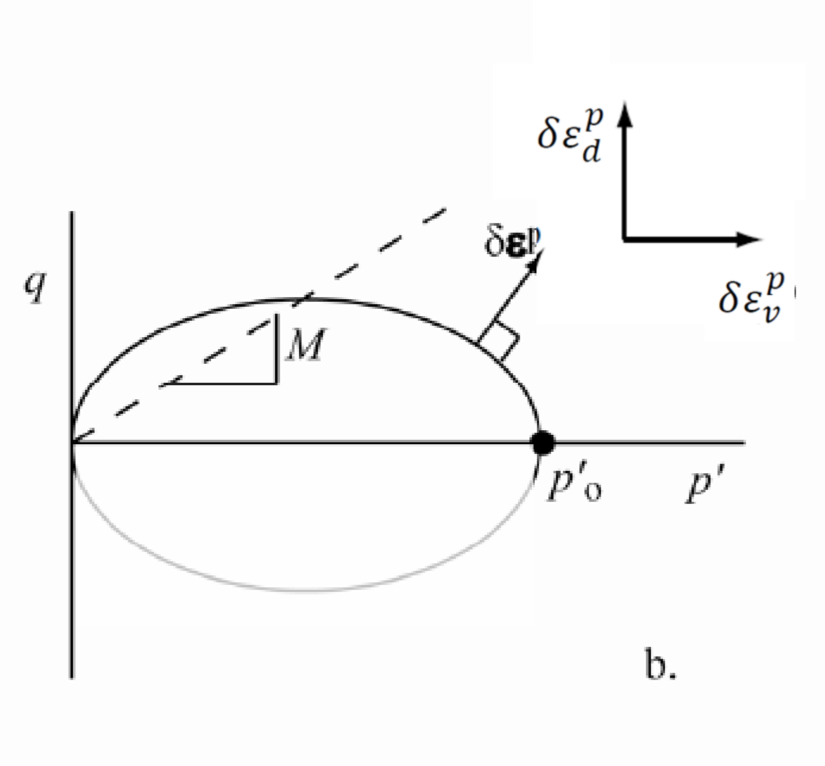
$$\begin{bmatrix} d\varepsilon_v^e \\ d\varepsilon_d^e \end{bmatrix} = \begin{bmatrix} {}^{1}/_{K} & 0 \\ 0 & {}^{1}/_{3G} \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix}$$

From MCC

$$\begin{bmatrix} d\varepsilon_v^e \\ d\varepsilon_d^e \end{bmatrix} = \begin{bmatrix} {}^{\kappa}/_{vp'} & 0 \\ 0 & {}^{1}/_{3G} \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix}$$

MCC – Yield surface

MCC in q-p' plane



Wood, 1990

Effective preconsolidation pressure

$$F = q^2 - M^2\left[p'(p'_0 - p')\right] = 0$$

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2}$$

$$\eta = \frac{q}{p'}$$

MCC – Plastic potential and flow rule

Associated flow rule

$$g = F = q^2 - M^2[p'(p'_0 - p')] = 0$$

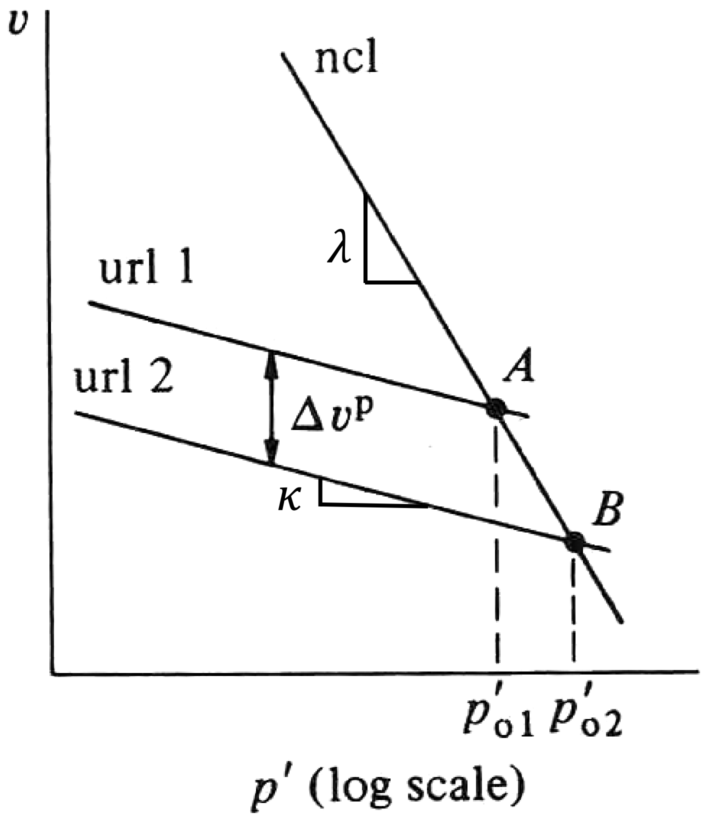
Volumetric and deviatoric plastic strain

$$d\varepsilon_v^p = \mu \frac{\partial g}{\partial p'}$$

$$d\varepsilon_d^p = \mu \frac{\partial g}{\partial q}$$

MCC – Hardening rule

Successive unloading-reloading paths



Wood, 1990

$$\Delta \upsilon^p = \Delta \upsilon - \Delta \upsilon^e = -(\lambda - \kappa) \ln\left(\frac{p'_{02}}{p'_{01}}\right)$$

$$d\upsilon^p = -(\lambda - \kappa)\frac{dp'_0}{p'_0}$$

$$d\varepsilon_v^p = (\lambda - \kappa)\frac{dp'_0}{\upsilon p'_0}$$

$$dp'_0 = \frac{\upsilon p'_0}{\lambda - \kappa} d\varepsilon_v^p$$

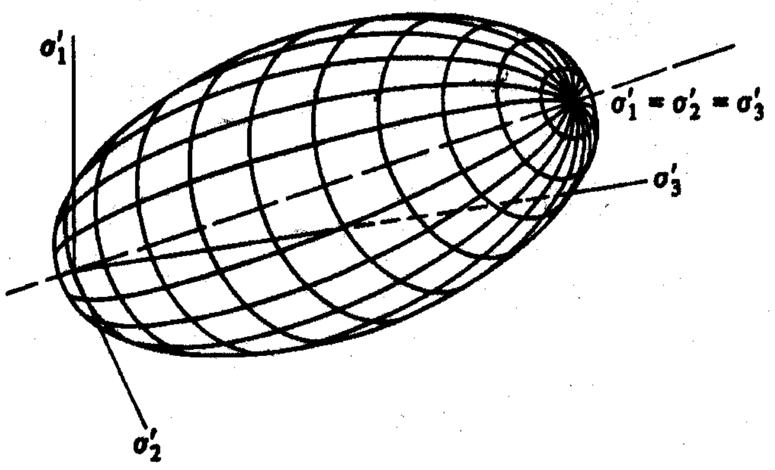
MCC – Summary

i. Elastic part $d\varepsilon_\upsilon^e = \dfrac{\kappa}{\upsilon p'} dp'$ $\quad d\varepsilon_d^e = \dfrac{dq}{3G}$

ii. Yield function $F = q^2 - M^2\left[p'(p_0' - p')\right] = 0$

iii. Plastic potential $g = F = q^2 - M^2\left[p'(p_0' - p')\right] = 0$

iv. Hardening rule $\dfrac{dp_0'}{p_0'} = \dfrac{\upsilon}{\lambda - \kappa} d\varepsilon_v^p$



MCC parameters

- Elastic: κ, G
- Plastic: M, λ, Γ

MCC – Summary

Full plastic compliance relationship

$$\begin{pmatrix} \delta\varepsilon_v^p \\ \delta\varepsilon_d^p \end{pmatrix} = \frac{\lambda-\kappa}{vp'(M^2+\eta^2)} \begin{pmatrix} M^2-\eta^2 & 2\eta \\ 2\eta & \dfrac{4\eta^2}{M^2-\eta^2} \end{pmatrix} \begin{pmatrix} \delta p' \\ \delta q \end{pmatrix}$$

Hardening modulus → $H = -\frac{\partial F}{\partial p_0'}\frac{\partial p_0'}{\partial \varepsilon_p^p}\frac{\partial g}{\partial p'} = -(-p')(\frac{vp_0'}{\lambda-\kappa})(2p'-p_0')$

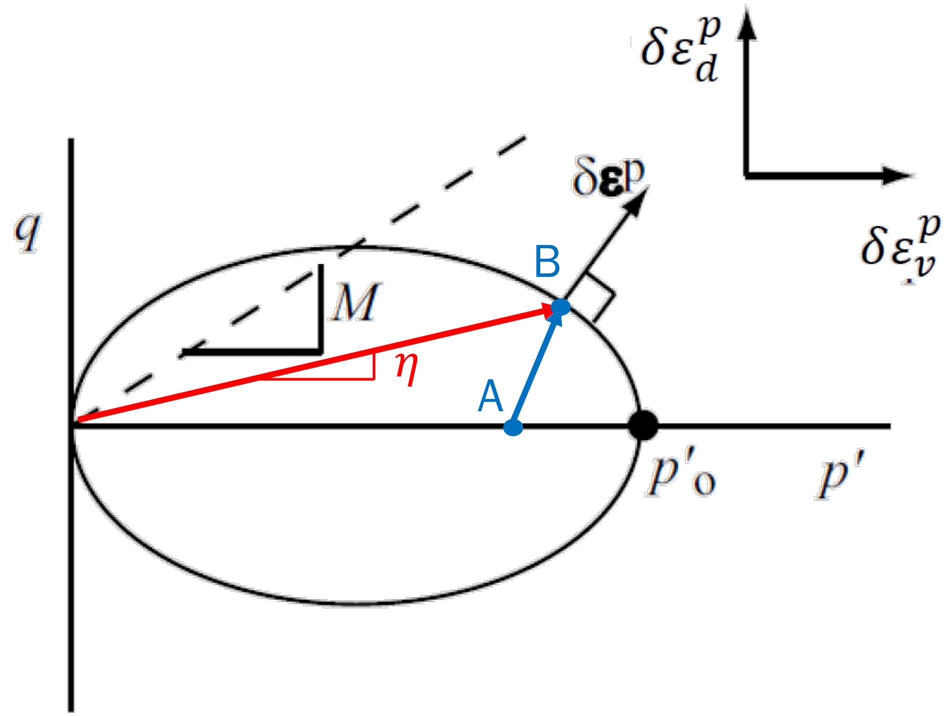
Full elasto-plastic stiffness relationship

$$\begin{pmatrix} \delta p' \\ \delta q \end{pmatrix} = \left[\begin{pmatrix} \dfrac{vp'}{\kappa} & 0 \\ 0 & 3G \end{pmatrix} - \frac{\begin{pmatrix} \left(\dfrac{vp'}{\kappa}\right)^2 (2p'-p_0')^2 & \dfrac{6Gvp'q(2p'-p_0')}{M^2\kappa} \\ \dfrac{6Gvp'q(2p'-p_0')}{M^2\kappa} & \dfrac{36G^2q^2}{M^4} \end{pmatrix}}{\dfrac{vp'}{\kappa}(2p'-p_0')^2 + \dfrac{12Gq^2}{M^4} + \dfrac{vp'p_0'(2p'-p_0')}{\lambda-\kappa}} \right] \begin{pmatrix} \delta\varepsilon_v \\ \delta\varepsilon_d \end{pmatrix}$$

MCC Prediction

MCC - Plastic strains increments

The plastic strain increments are given as usual by the partial derivatives of the plastic potential function



$$\begin{pmatrix} \delta\varepsilon_v^p \\ \delta\varepsilon_d^p \end{pmatrix} = \mu \begin{pmatrix} \dfrac{\partial g}{\partial p'} \\ \dfrac{\partial g}{\partial q} \end{pmatrix} = \mu \begin{pmatrix} 2p' - p'_0 \\ \dfrac{2q}{M^2} \end{pmatrix}$$

The ratio provides the direction of the plastic strains:

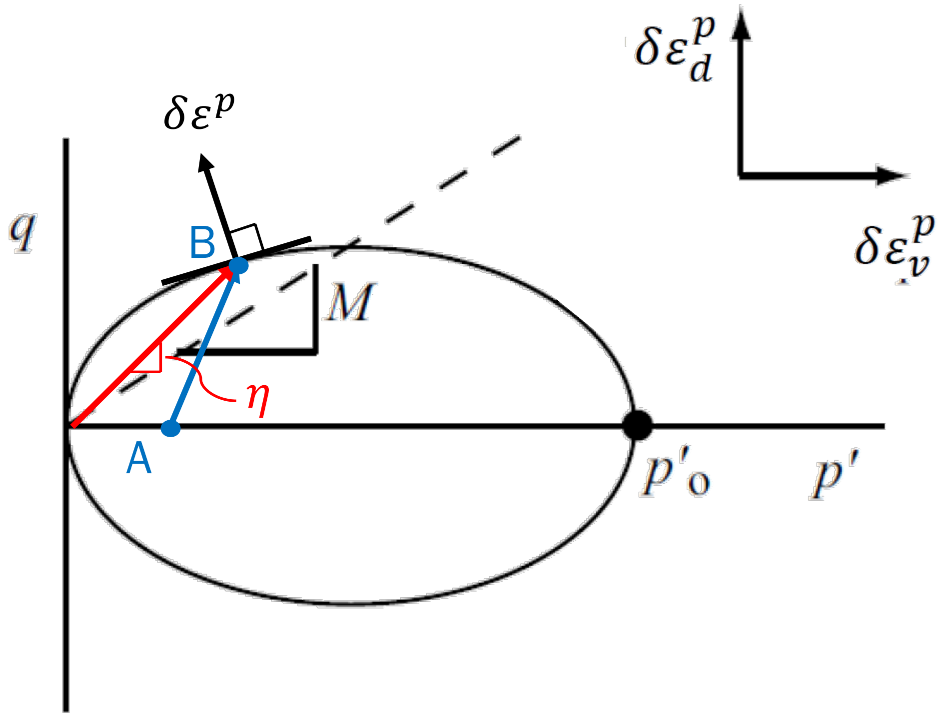
$$\frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} = \frac{M^2 - \eta^2}{2\eta}$$

$$\eta = \frac{q}{p'} < M \Longrightarrow \frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} > 0$$

<u>Compression plus distorsion</u>

MCC - Plastic strains increments

The plastic strain increments are given as usual by the partial derivatives of the plastic potential function



$$\begin{pmatrix} \delta\varepsilon_v^p \\ \delta\varepsilon_d^p \end{pmatrix} = \mu \begin{pmatrix} \dfrac{\partial g}{\partial p'} \\ \dfrac{\partial g}{\partial q} \end{pmatrix} = \mu \begin{pmatrix} 2p' - p'_0 \\ \dfrac{2q}{M^2} \end{pmatrix}$$

The ratio provides the direction of the plastic strains:

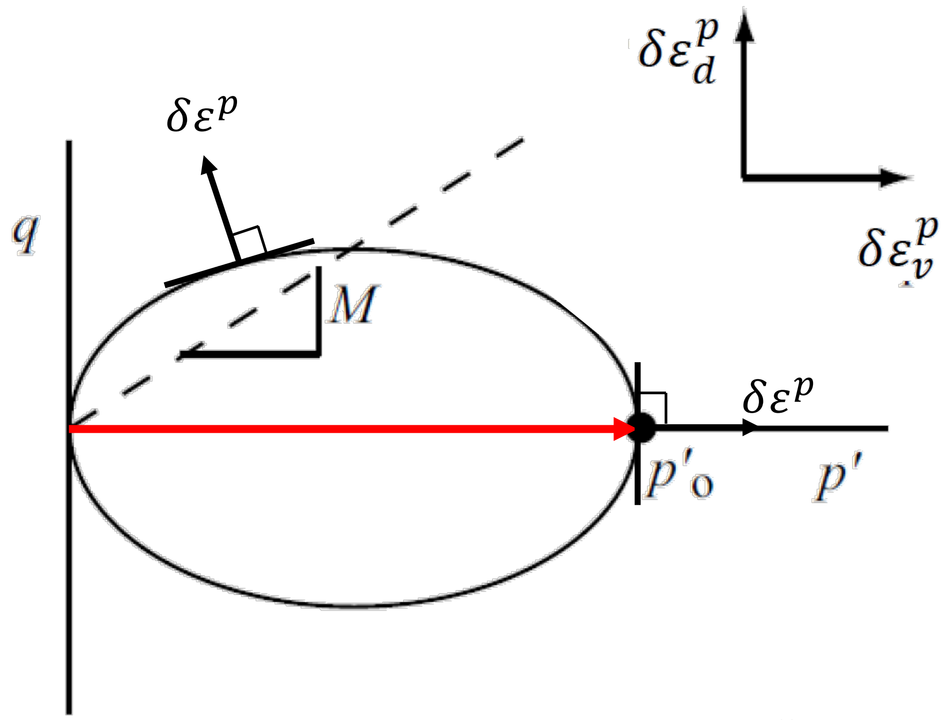
$$\frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} = \frac{M^2 - \eta^2}{2\eta}$$

$$\eta > M \Longrightarrow \frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} < 0$$

<u>Expansion (dilation) plus distorsion</u>

MCC - Plastic strains increments

The plastic strain increments are given as usual by the partial derivatives of the plastic potential function



$$\begin{pmatrix} \delta\varepsilon_v^p \\ \delta\varepsilon_d^p \end{pmatrix} = \mu \begin{pmatrix} \dfrac{\partial g}{\partial p'} \\ \dfrac{\partial g}{\partial q} \end{pmatrix} = \mu \begin{pmatrix} 2p' - p'_0 \\ \dfrac{2q}{M^2} \end{pmatrix}$$

The ratio provides the direction of the plastic strains:

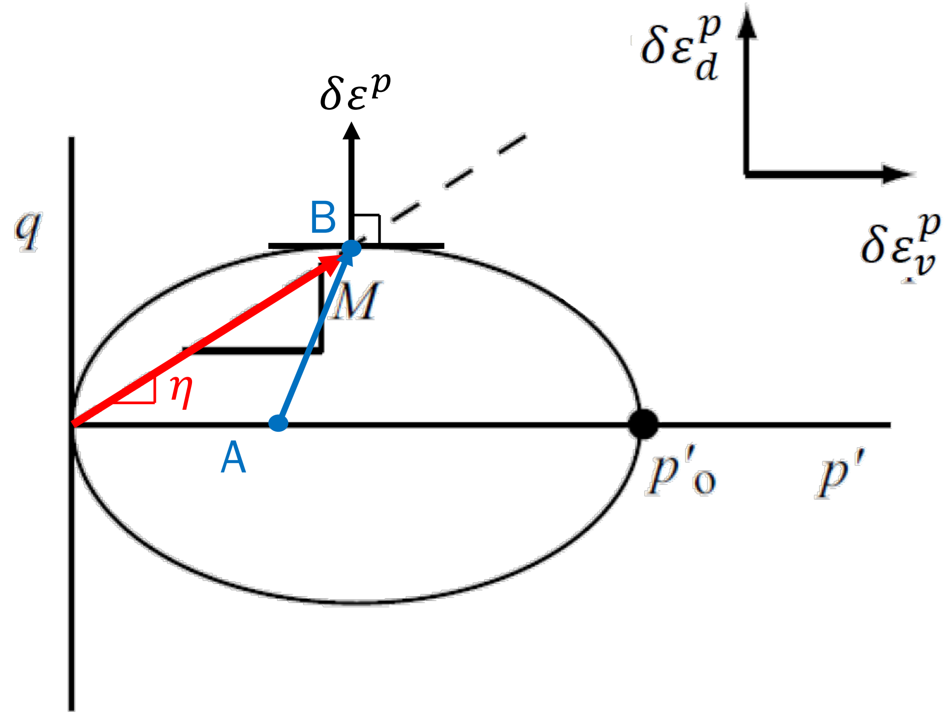
$$\frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} = \frac{M^2 - \eta^2}{2\eta}$$

$$\eta = 0 \Longrightarrow \frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} \to \infty$$

<u>Compression without distortion</u> (isotropic compression)

MCC - Plastic strains increments

The plastic strain increments are given as usual by the partial derivatives of the plastic potential function



$$\begin{pmatrix} \delta\varepsilon_v^p \\ \delta\varepsilon_d^p \end{pmatrix} = \mu \begin{pmatrix} \dfrac{\partial g}{\partial p'} \\ \dfrac{\partial g}{\partial q} \end{pmatrix} = \mu \begin{pmatrix} 2p' - p'_0 \\ \dfrac{2q}{M^2} \end{pmatrix}$$

The ratio provides the direction of the plastic strains:

$$\frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} = \frac{M^2 - \eta^2}{2\eta}$$

$$\eta = M \Longrightarrow \frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} = 0$$

<u>Distorsion without compression</u> (critical state conditions)

MCC - Plastic strains increments

What happens as η tends to M?

- The increments of plastic volumetric strains becomes smaller and smaller
- As a consequence, the evolution of the hardening parameter tends to zero
- The shear compliance tends to infinity (shear stiffness tends to zero)

Tends to zero

$$\begin{pmatrix} \delta\varepsilon_v^p \\ \delta\varepsilon_d^p \end{pmatrix} = \frac{\lambda - \kappa}{vp'(M^2 + \eta^2)} \begin{pmatrix} M^2 - \eta^2 & 2\eta \\ 2\eta & \dfrac{4\eta^2}{M^2 - \eta^2} \end{pmatrix} \begin{pmatrix} \delta p' \\ \delta q \end{pmatrix}$$

Tends to infinity

MCC - Plastic strains increments

What happens as η tends to M?

Tends to zero

$$\begin{pmatrix} \delta\varepsilon_v^p \\ \delta\varepsilon_d^p \end{pmatrix} = \frac{\lambda - \kappa}{vp'(M^2 + \eta^2)} \begin{pmatrix} M^2 - \eta^2 & 2\eta \\ 2\eta & \dfrac{4\eta^2}{M^2 - \eta^2} \end{pmatrix} \begin{pmatrix} \delta p' \\ \delta q \end{pmatrix}$$

Tends to infinity

- Asymptotic perfectly plastic condition shear strain continue without any change in yield locus size, stresses and volumetric strain

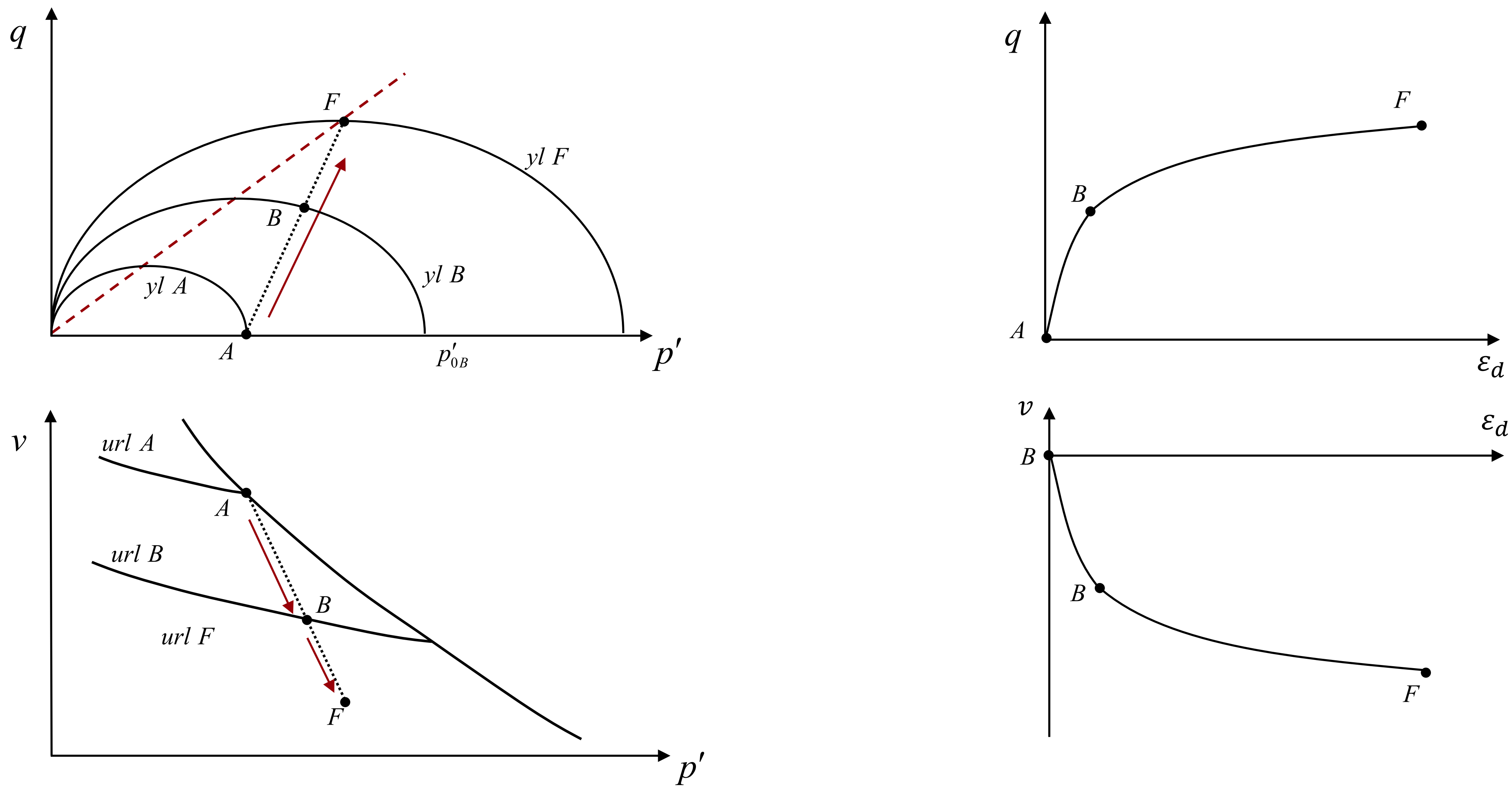
$\longrightarrow$ Critical state

$$\eta \to M \Longrightarrow \delta\varepsilon_v^p \to 0 \Longrightarrow \delta p' \to 0$$

$$\eta \to M \Longrightarrow \frac{\delta\varepsilon_d^p}{\delta q} \to \infty$$

MCC – Prediction

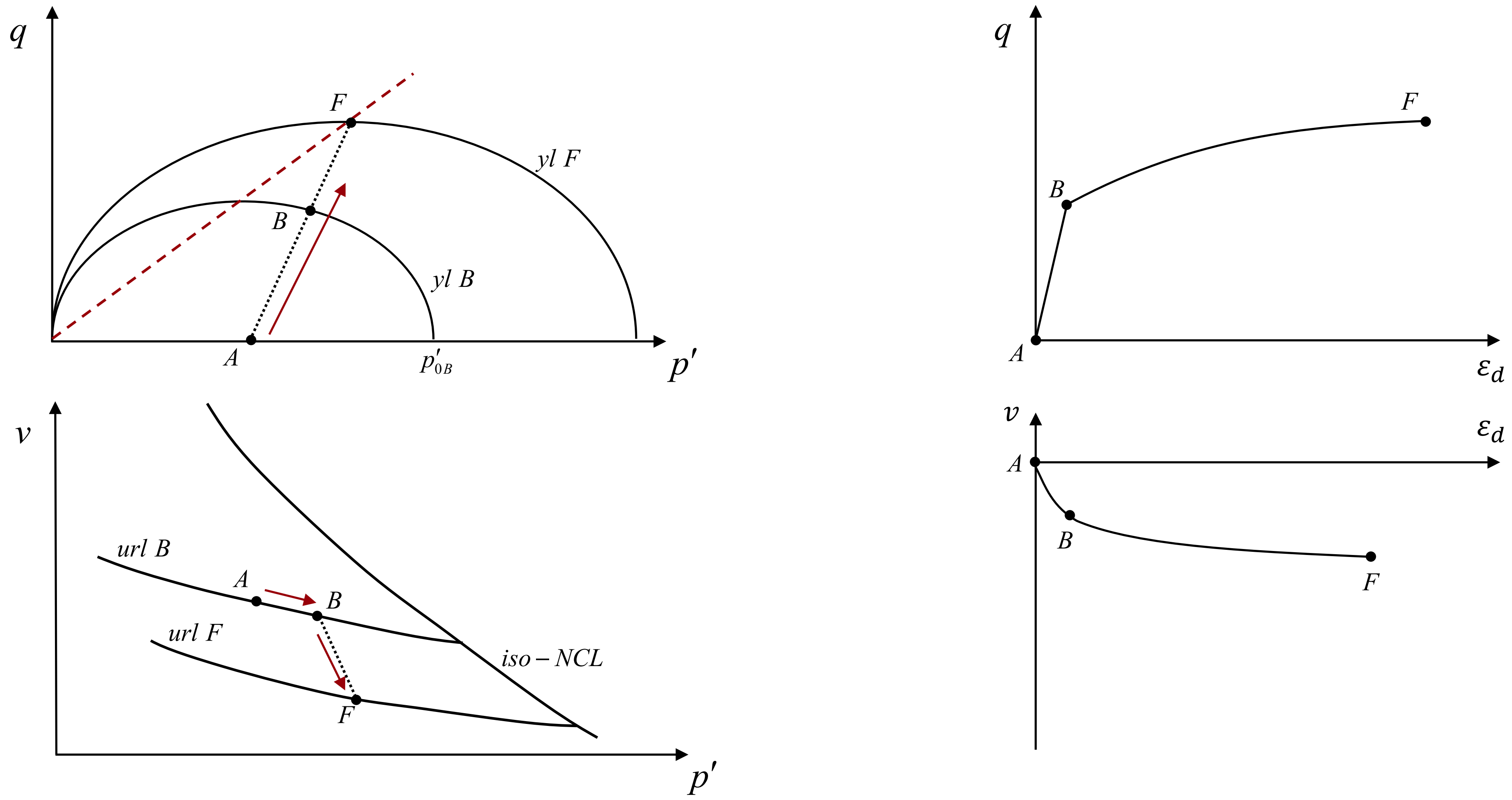
Drained conventional triaxial compression test – Normally consolidated



Wood, 1990

MCC – Prediction

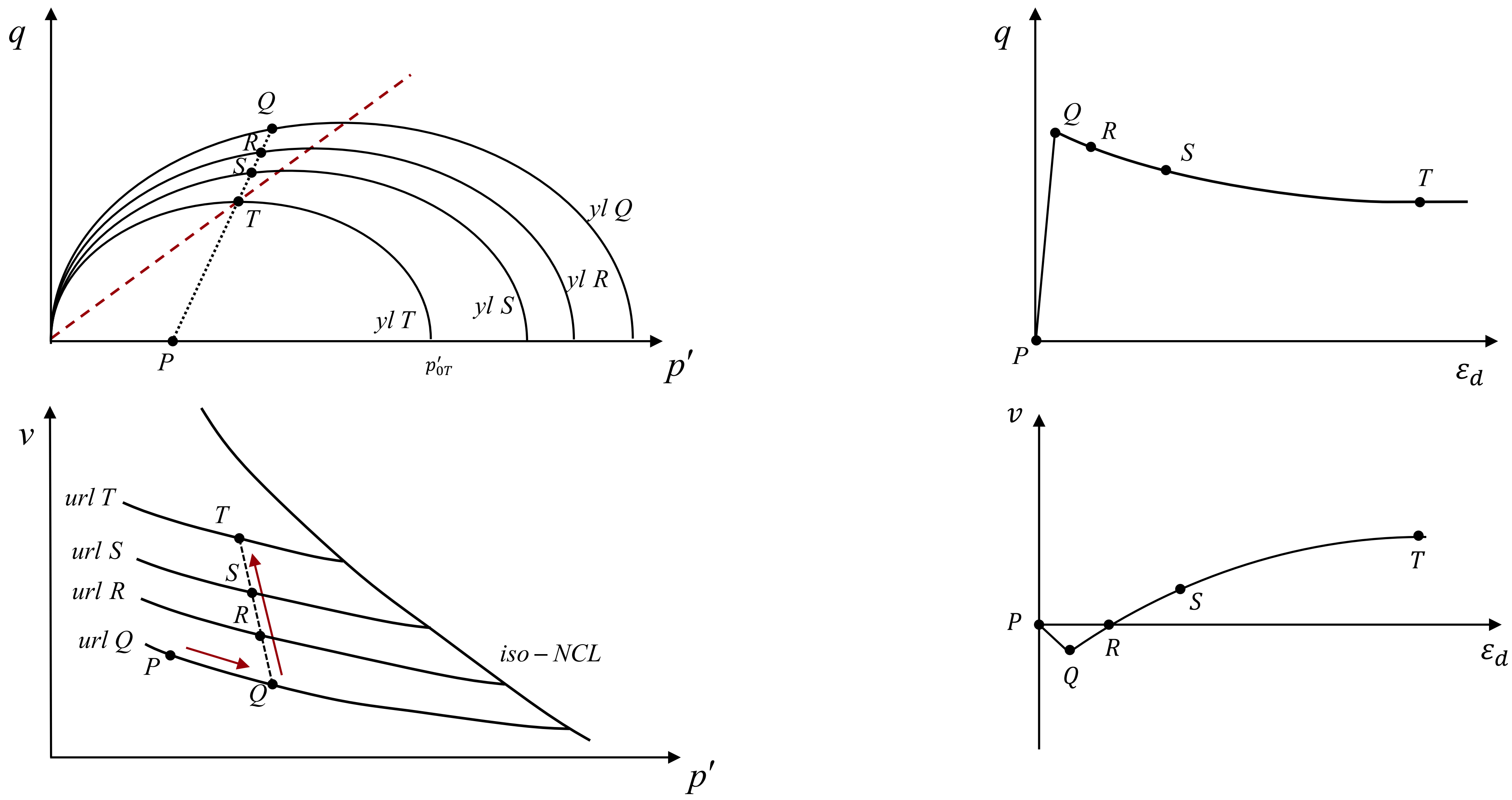
Drained conventional triaxial compression test – Lightly over consolidated



Wood, 1990

MCC – Prediction

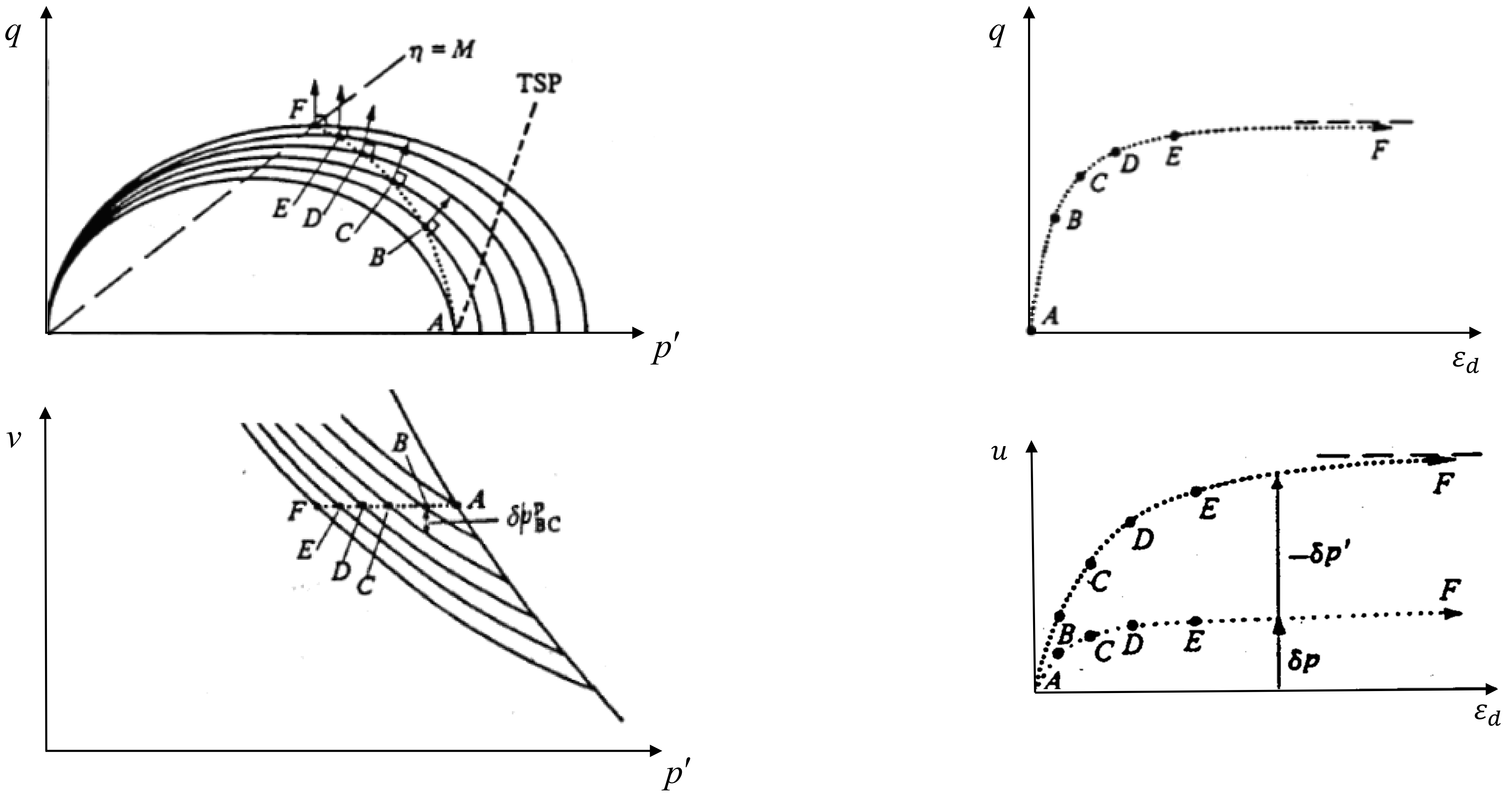
Drained conventional triaxial compression test – Heavily over consolidated



Wood, 1990

MCC – Prediction

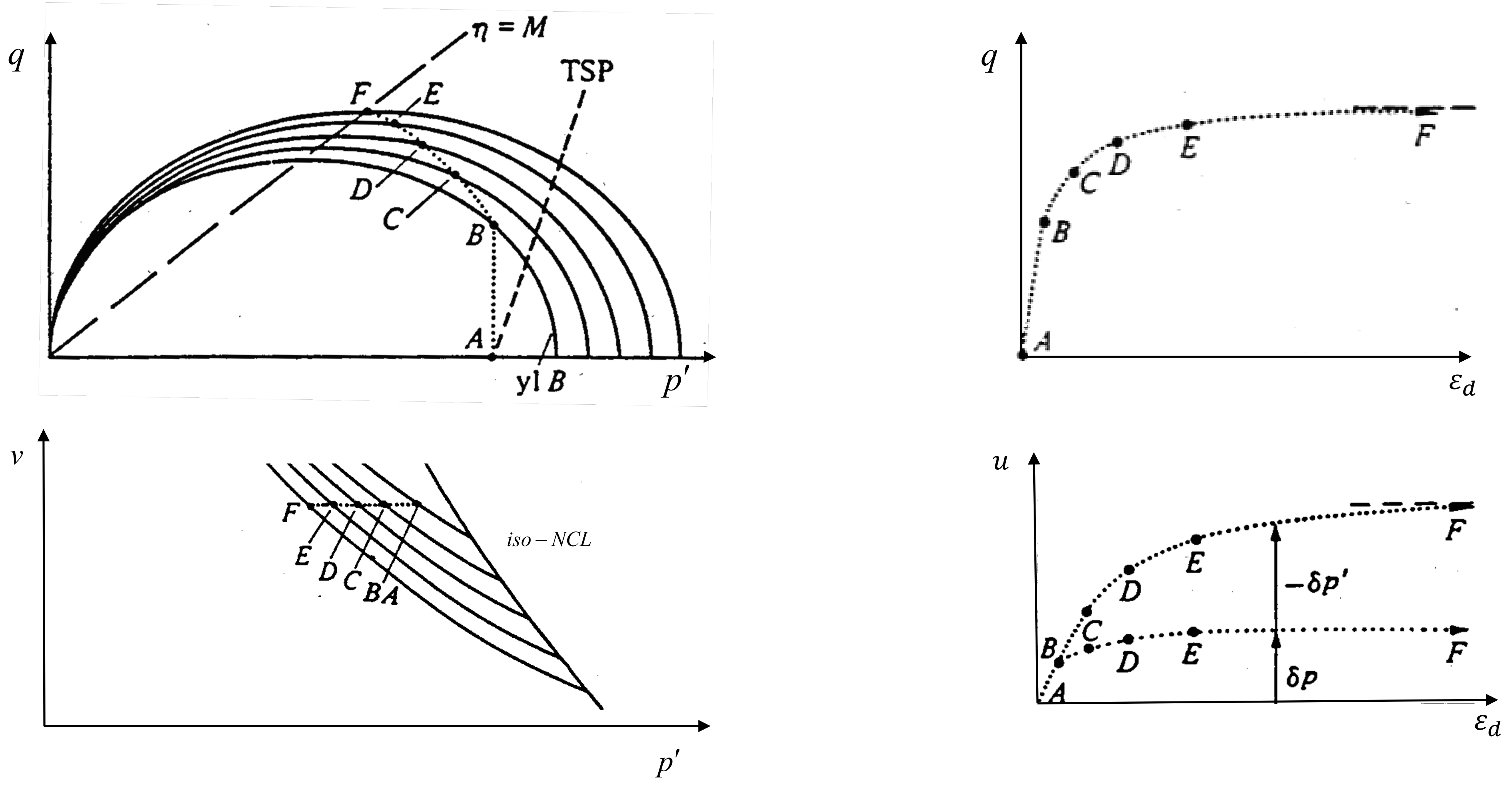
Undrained conventional triaxial compression test – Normally consolidated



Wood, 1990

MCC – Prediction

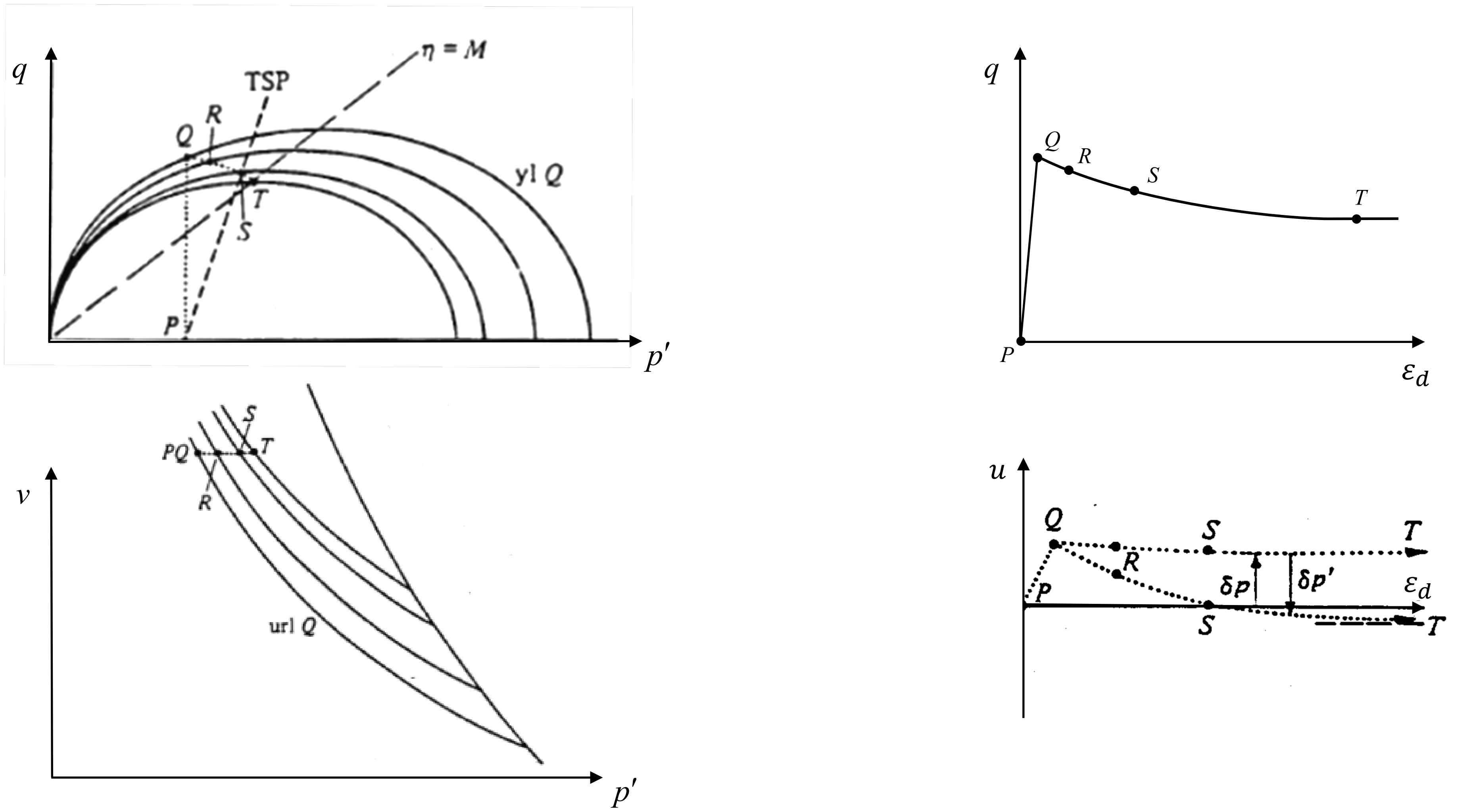
Undrained conventional triaxial compression test - Lightly over consolidated



Wood, 1990

MCC – Prediction

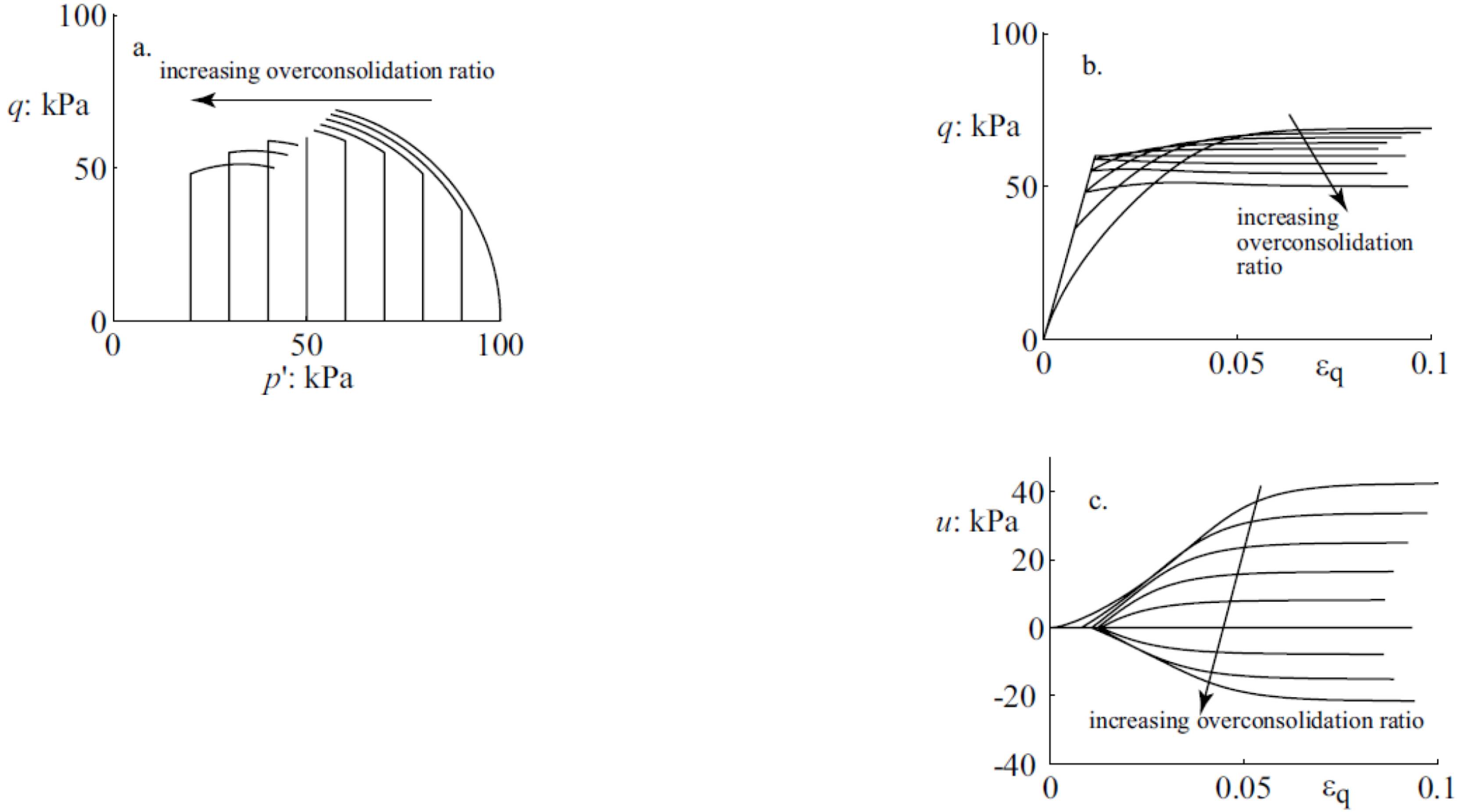
Undrained conventional triaxial compression test - Heavily over consolidated



Wood, 1990

MCC – Prediction

Undrained conventional triaxial compression test – From lightly to heavily over consolidated



Conclusion

Conclusion

Critical state :

State in which the soil reaches its ultimate shear strength; the shearing can continue without any tendency of the soil to change its volume

Modified Cam-Clay :

Modelling of soil behavior within the framework of hardening elasto-plasticity

Estimation of elastic and plastic deformation using MCC

MCC can be used for hand calculations of elements undergoing simple modes of deformation

References

1. Roscoe, K.H., Shofield, A., & Worth, C.P. (1958) « On the yielding of soils », Geotechnique, Vol. 8, pp. 22-53
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6. Atkinson, J. H. and Bransby, P. L. (1978). The Mechanics of Soils, An Introduction to Critical State Soil Mechanics. McGraw-Hill, London.

Thank you for your attention